

A Structural Analysis Approach to Residential Foundation Performance Evaluation

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Abstract: This paper uses fundamental structural mechanics to develop a logical and rational basis for making a first-order engineering estimate of the damage that could result to a brick veneer single family home as a result of climate related foundation movement. It is assumed that the damage to the house results from the curvature of the foundation resulting from climate related foundation movement. The building elements analyzed include a nonreinforced brick veneer wall, a masonry fireplace chimney, a door frame and a wall covered with gypsum board. The methods used rely on estimates of foundation deflection profile geometry, the geometry of the supported structure, strain superposition, and critical strains of the brick veneer and gypsum board wall coverings. Quantitative estimates of damage to the house can serve as a logical and rational basis for evaluating the performance of the foundation.

Introduction

Homeowners and buyers of homes that exhibit damage caused by slab-on-ground foundation movement may face the question of whether the foundation should be underpinned to mitigate future movement. Engineers are often retained to offer advice as to whether underpinning is a useful option. Some engineers rely heavily on rudimentary elevation surveys; other engineers may use damage criteria such as drywall and brick veneer cracking and door frame distortion.² Neither of these approaches are related in a rational and logical way to the design and construction of a post-tensioned slab-on-ground foundation. Also, they do not typically provide a rational quantitative criterion for making a recommendation to underpin, or not underpin, the foundation.

The current paper uses fundamental structural mechanics concepts (specifically geometry of deformation and material behavior) to understand and describe how the deflection of a slab-on-ground foundation on expansive soils can damage the supported structure. Simple structural engineering models are presented which allow a rational, quantitative first-order engineering estimate of damage that could be inflicted on a brick veneer home with interior wood frame walls covered with gypsum board in the form of cracks when the walls are supported by a slab-on-ground foundation on expansive soils. Structural engineering models are also presented that permit the analysis of door frame distortion and fireplace chimney rotation due to deflection of a slab-on-ground foundation on expansive soils. These models have several practical applications as listed below:

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² K. B. Bondy, (2000). "Performance evaluation of residential concrete foundations." Technical Notes, 9, 1-6

- The structural models can be used to estimate the damage a homeowner or purchaser of a home founded on a slab-on-ground foundation might expect in the future. Being able to use an engineering model to provide a prospective buyer with quantitative estimates of the amount damage the walls could experience, how susceptible to cracking the wall coverings are, how susceptible the doors could be to binding or sticking and how much the fireplace chimney might be expected rotate away from the house can allow a potential buyer to make a more informed decision as to whether to purchase the home. Also, if a decision were made to purchase the home, the buyer would have a more realistic understanding of what foundation-induced damage to expect.
- Current methods of foundation design such as the PTI method are only a partial solution to foundation design. A more complete foundation design would include an estimate of the damage that foundation movement could cause the supported structure.³ The structural analysis tools used in this paper show one way foundation related damage can be estimated.
- A rational quantitative understanding of how differential deflection of a slab-on-ground foundation damages the supported structure can provide a rational, unbiased basis for evaluating the performance of an existing foundation. This type of evaluation does not depend on elevation measurements.
- Existing, visible damage to a house can be compared to damage that might occur if the foundation were underpinned. Most foundation repair companies will warrant that future movement of underpinned areas of the foundation will deflect no more than 1-inch for each 360-inches of span. This means nothing to the average homeowner. The engineering models we present can be used to estimate the damage that could result from foundation movement corresponding to a deflection ratio of 1/360; and the estimated damage reported terms that are meaningful to a homeowner. The damage estimate then can be compared to the existing observable damage allowing a more rational decision concerning whether a foundation should be underpinned.

A Brief Description of the Structural Analysis Approach to Foundation Performance Evaluation

The specific methods we use are based, in part, on an engineering model of foundation distortion similar to the model commonly used in the design of slab-on-grade foundations. For example, the design procedure described in the publication, **Design and Construction of Post-Tensioned Slabs-on-Ground published by the Post-Tensioning Institute (PTI)**⁴ uses two models of slab-

³ Burland, J.B. and Wroth, C.P. (1975); *Settlement of Buildings and Associated Damage*, Building Research Establishment Current Paper, 33(75), Building Research Establishment, Watford England, pp. 626-628

⁴ **Design and Construction of Post-Tensioned Slabs-on-Ground**, Post-Tensioning Institute, Phoenix, Arizona, 1996; Professor Robert Lytton has identified four principle modes of plate bending and distortion that could be applied to slab-on-ground foundations (center lift

on-ground distortion; a distortion mode called center lift, which could more accurately be described as “edge fall”⁵, and a second distortion mode called edge lift. In the center lift or edge fall mode, the foundation perimeter sags downward while the central area of the foundation remains stable. This distortion mode is the likely result when the weather is very dry and the soil supporting the perimeter of the foundation dries and shrinks allowing the perimeter of the slab-on-ground foundation to sag. In the edge lift distortion mode, the perimeter of the foundation lifts up as the supporting soil at the slab perimeter wets up and swells causing the perimeter of the foundation to lift. The center lift distortion mode is more damaging, at least in terms of brick veneer cracking, to the supported structure.⁶ For this reason, the approach we use to estimate the damage that could be done to the supported structure by foundation movement uses a model based on a center lift distortion mode.

The following is a summary of the approach that we take:

1. The first step is to create a geometric model of the foundation in a center lift distortion mode. The model must meet several requirements in addition to having a deflection surface that corresponds to an center lift distortion shape as listed below:
 - We describe the shape of the deflection surface in terms of a set of deflection curves. No deflection curve in the model is allowed to have a deflection ratio greater than what has been specified for purposes of the analysis such as 1/360.
 - The overall distortion profile is described in terms of what we refer to as primary deflection curves. The primary deflection curves are developed in a way that insures that they are geometrically compatible with one another.
 - Secondary deflection curves are also developed so that the deflections and slopes at any location on the foundation can be calculated. The secondary deflection curves are developed in a manner so that they are geometrically compatible with the assumed foundation distortion mode and also geometrically compatible with the primary deflection curves.
 - Since the foundation is normally much longer and wider than it is deep, shear distortion can be ignored. The deflection curves can therefore be modeled using a

cylindrical, edge lift cylindrical, center heave for perimeter drying and center heave for diagonal drying) in Lytton, Robert L., *Design Criteria for Residential Slabs and Grillage Rafts on Reactive Clay*, **Report for the Australian Commonwealth Scientific and Industrial Research Organization, Division of Applied Geomechanics**, Melbourne, Australia, November 1970.

⁵ K. B. Bondy, (2000). “Performance evaluation of residential concrete foundations.” Technical Notes, 9, 1-6

⁶ Burland, J.B. and Wroth, C.P. (1975); *Settlement of Buildings and Associated Damage*, Building Research Establishment Current Paper, 33(75), Building Research Establishment, Watford England, pp. 626-628

Euler-Bernoulli beam model subjected to pure bending.⁷ We show how this is done using cantilever beam models in an example in the section titled **Geometry of Foundation Deformation**.

2. Using published values, we estimate the critical extensional, shear and combined strains for each wall covering. (The critical strains are the strains that result in visible, observable cracking in the wall covering.⁸)
3. The brick veneer walls are modeled as deep beams in which shear deflection must be considered. The maximum bending deflection and shear deflection is estimated for brick veneer walls. An engineering estimate is made of the bending strain, the shear strain and the combined strain in brick veneer walls. If the brick veneer wall is judged to be likely to crack or fracture due to foundation movement, elastic beam and geometric models are used to estimate the cumulative width of the brick veneer cracks.
4. Gypsum board covered walls and door frames are modeled as frame structures subjected to an angular distortion calculated from the deflection curves developed in step 1. Door frames are analyzed for binding and gypsum board covered walls are analyzed for cracking.
5. Fireplace chimneys are modeled as rigid columns that rotate due to bending of the foundation using the deflection curve or curves developed in step 1. Fireplace chimneys are analyzed for rotation away from the house frame.

Using Structural Mechanics to Relate Observed Damage to Foundation Movement – Background

There has been some interesting published work where structural mechanics models were used in an attempt to relate damage in the supported structure to foundation movement. The most important of these papers are summarized below:

⁷ **Mechanics of Materials**, 3rd edition, James M. Gere and Stephen P. Timoshenko, PWS-KENT Publishing Company, Boston, Massachusetts, p. 253

⁸ Polshin, D. E. and Tokar, R. A. (1957). *Maximum Allowable Non-Uniform Settlement of Structures*, **Proceedings, 4th International Conference on Soil Mechanics and Foundation Engineering.**, Vol 1, Butterworth's Scientific, London, England, 402-405.

Maximum Allowable Non-uniform Settlement of Structures, by D.E. Polshin and R.A. Tokar:⁹

This paper discussed two concepts that are central to any attempt to analytically relate foundation movement to observed damage in the supported structure; the concept of "critical strain" and the recognition that the ratio of the length to the height of the supported building is fundamentally important to relating foundation movement to masonry damage in the supported structure. The "critical strain" of a building material is simply the strain at which damage to the material first becomes visible in the form of observable cracks. Polshin and Tokar presented a theoretical limiting relationship between differential foundation deflection and the ratio of the building length and height that described when cracking in load bearing masonry walls would be visible. The paper also presented some empirical studies of both cracked and uncracked buildings. The empirical measurements were compared to the calculations; the calculations were found to be in reasonable agreement with the measurements.

Settlement of Buildings and Associated Damage, by J.B. Burland and C.P. Wroth:¹⁰

The importance of this paper cannot be overstated. In this paper, Burland and Wroth proposed an analytical model for calculating the foundation deflection that would result in visible cracking in the load-bearing masonry and masonry infill building walls. Building on the work by Polshin and Tokar, Burland and Wroth developed a procedure using a beam analogy where the building was modeled as a "uniform, weightless, elastic beam of length L, height H and unit thickness". The model assumed that the deflected shape of the soffit of the beam was known. Since, for the typical building, the ratio of wall height to the wall length is relatively large, the potential for significant shear distortion in the walls was considered. Using deep beam theory, the beam model calculates how much of the beam or wall distortion is due to shear and how much is due to pure bending. The shear strain and the bending strain is then combined to calculate the combined strain. The calculated strains can then be compared to the critical shear, bending and combined strains for the building materials in order to determine if the masonry wall will fracture.

Tolerable Settlement of Buildings, by Harvey E. Wahls¹¹:

The purpose of this paper was to review published "concepts and practices for establishing tolerable settlements for buildings." Wahls observes that "critical settlements have not been determined analytically" due to the complexity of the problem. He concludes that the "beam analogy proposed by Burland and Wroth provides a conceptual understanding of the factors that control tolerable settlement"... The beam analogy "can be used to identify information required

⁹ Polshin, D. E. and Tokar, R. A. (1957). *Maximum Allowable Non-Uniform Settlement of Structures*, **Proceedings, 4th International Conference on Soil Mechanics and Foundation Engineering**, Vol 1, Butterworth's Scientific, London, England, 402-405.

¹⁰ Burland, J.B. and Wroth, C.P. (1975), *Settlement of Buildings and Associated Damage*, Building Research Establishment Current Paper, 33(75), Building Research Establishment, Watford England, pp. 626-628

¹¹ Harvey E. Wahls, *Tolerable Settlement of Buildings*, Journal of the Geotechnical Engineering Division, ASCE, Vol. 107, November 1981

for meaningful interpretation of (empirical) case studies and to separate data from the case studies into appropriate categories of structures."

Wahls also stated the following concerning existing empirical case studies: "Several inherent limitations to (the) empirical approach to tolerable settlements should be recognized. The primary problem is the scarcity of complete and reliable case studies. The assessment of the type and degree of damage usually is qualitative. Often, detailed descriptions of the structures and soil conditions are not available. Sometimes erroneous assumptions are made regarding the relation of damage to settlementRelatively few settlements are reported for undamaged buildings, and, consequently, the available case studies are biased toward situations in which damage has been observed."

Building Response to Excavation Induced Settlement, by Marco D. Boscardin and Edward J. Cording¹²

This paper presented an analytical model to estimate the response of buildings that could potentially be damaged as a result of horizontal and vertical ground movements resulting from adjacent construction projects. In essence, Boscardin and Cording brought together several ideas found in the above referenced work by Burland and Wroth and by Polshin and Tokar to address the practical problem of quantitatively estimating the degree of damage to masonry wall structures and small frame structures adjacent to excavations. They emphasized the importance of considering horizontal ground movements. The angular distortion a building was likely to experience due to tunneling excavations was related graphically to the position of a building relative to the tunnel settlement trough. It was implied that for tunnel-related building distortion the degree of damage to a building could be assessed using various graphs with estimates of building settlement, angular distortion, and horizontal ground movements. The results of the analytical model were compared to field measurements.

Where Wahls saw only a limited role for the use of analytical methods of analysis in determining the allowable calculated deflection, Boscardin and Cording concluded that the use of analytical models "appeared to have promise" for predicting cracking in the walls of bearing masonry wall structures adjacent to excavations.

Ground-Movement Related Building Damage, by Storer J. Boone:¹³

This is an extraordinarily interesting paper that brought together several ideas found in the above referenced work by Polshin and Tokar, Burland and Wroth, and Boscardin and Cording to address a practical problem. Boone noted that various construction projects can damage adjacent structures. This is particularly true for tunneling under or near existing buildings. Large construction projects in developed areas such as cities can result in potentially damaging

¹² Boscardin, M. D., and Cording, E. J. (1989). *Building Response to Excavation-Induced Settlement*, Journal Geotechnical Engineering, ASCE, 115(1), 1-21.

¹³ Boone, Storer J. (1996). *Ground-Movement-Related Building Damage*, Journal of Geotechnical Engineering, ASCE, 122(11), 886-896.

numerous structures including buildings.¹⁴ Boone stated that "intensive evaluations for each structure that might be impacted by construction becomes both impractical and uneconomical for tunneling or large open-cut excavations. Often, a simple and conservative method is required in the first round of assessments to narrow detailed studies to those structures that are judged to be critical." Boone proposed a step by step method of damage assessment that provided "a logical approach to evaluating ground-movement-related damage." The method proposed by Boone allowed the following:

“

1. Consideration of building geometry
2. Consideration of building materials
3. Consideration of ground-deformation directions and geometry
4. Calculation of quantifiable results that bear physical meaning with respect to both the structure and the ground
5. Categorization of potential damage
6. Determination of “allowable settlements” for new construction
7. Relatively rapid building damage assessments for construction next to existing structures”¹⁵

The Boone procedure included the use of fundamental structural mechanics, specifically including deep beam theory and plane-strain mechanics, to determine if anticipated ground movements resulting from tunneling excavation or large open-cut excavations would result in cracks in load-bearing masonry walls and masonry infill walls. For those buildings that were predicted to have cracked walls, single equivalent wall cracks were estimated. The single equivalent wall cracks were then used with a published table that relates wall crack widths to six damage categories ranging from negligible to very severe. In this way, Boone’s approach provided a rational and logical method for estimating the severity of damage likely to be caused to a building adjacent to a tunneling or open-cut excavation project.

It is clear that the issue addressed by Boone is very similar to the foundation performance issues facing homebuyers and the engineers they are relying on for advice. A purchaser of a new home has no historical record of how the foundation has performed in the past since there is no past. But a quantitative description of how the foundation could perform could be very useful for many buyers just as a quantitative description of damage that could result due to excavation adjacent to an existing foundation is useful to a contractor in quantifying his risks. If simple structural mechanics concepts can be used to develop a useful quantitative description of damage due to soil profile changes due to excavation, it seems reasonable that a structural mechanics approach could also be useful in developing a quantitative description of damage that could result to a single family home from foundation deflection.

There are also some clear differences between the problem Boone is addressing and the problem addressed in this paper. Boone starts with a specified ground distortion profile; we start with an

¹⁴ Karl Terzaghi and Ralph B. Peck, **Soil Mechanics in Engineering Practice, 2nd edition**, John Wiley & Sons, New York, New York, 1967, p. 570-580

¹⁵ S.J. Boone: *Ground-Movement-Related Building Damage: Closure*, J. of Geotech. and Geoenvironmental Eng., 1998, ASCE, 124(5), 463-465

assumed foundation distortion profile. Boone uses structural mechanics to estimate the damage the exterior masonry load-bearing walls and framed masonry infill walls will experience; we use structural mechanics to estimate the cumulative crack width each brick veneer wall will experience as a result of foundation distortion. Boone uses the calculated cumulative crack width with a published table to arrive at a more complete description of the damage likely to be done to the building as a result of the change in the soil profile; we use structural mechanics models to calculate whether doors will stick, drywall will crack and how much a fireplace chimney will rotate away from the house structure.

A General Comment Concerning the Use of Structural Mechanics Models for Foundation Performance Evaluation

The next several sections discuss in detail the structural models we use in the evaluation of the performance of slab-on-ground foundations. It is important to understand the role of structural models in performance evaluation. At the design stage the purpose of a structural mechanics model is to provide a logical, rational and quantitative basis for making an engineering judgment as to safety and adequacy of a proposed design. In an analogous manner, when evaluating the performance of an existing foundation, the purpose of a structural mechanics model is to provide quantitative information that is useful when making engineering judgments concerning the performance of the foundation. Structural models need to be realistic enough to realistically exhibit the expected behavior of the foundation; they also need to be simple enough to analyze. In developing the specific models we describe in this paper, it was our intent to use models that could be understood and profitably used by any competent practicing structural engineer.

Geometry of Foundation Deformation

All the structural models described in this paper are based on an assumed foundation distortion profile defined in terms of the shape of various deflection curves. In the approach we use, the first step is to specify an assumed maximum foundation deflection ratio as that term is used in the PTI manual. The maximum foundation deflection ratio that is specified depends on the purpose of the analysis. Below are several reasons why the selection of an appropriate deflection ratio requires careful engineering judgment:

- Normally, the deflection ratios that could be considered range from a very optimistic .8/480 to 1.5/480. A recognized characteristic of reinforced concrete as a structural material is that the load deflection behavior is highly variable and can be predicted only within a wide range.¹⁶ Careful, controlled laboratory studies of simply supported reinforced concrete beams have demonstrated that 90% of the specimens had measured deflections that were between 20% less to 30% greater than what an elastic beam model predicts due to ". . . the large variability of in the properties of the constituent materials . . . and the quality control exercised in . . . construction."¹⁷ Another 5% deflected more than 30% more than the elastic model prediction and another 5% deflected less than 20%

¹⁶ McCormac, Jack C., 1993, **Design of Reinforced Concrete**, 3rd edition, HarperCollins College Publishers, New York, New York, p. 145

¹⁷ *Control of Deflection in Concrete Structures*, ACI 435-95, American Concrete Institute, p. 3

less than the elastic model prediction. Thus the studies indicate that 95% of the elevated reinforced beams deflected no more than 30% greater than that predicted by an elastic beam model.¹⁸

- The American Concrete Institute recommends that designers should anticipate that the actual deflection of elevated reinforced concrete structural members may exceed what elastic deflection calculations indicate by 40%.¹⁹ This may be inadequate for single family home construction. In my opinion, 50% may be more realistic.
- The current PTI manual, **Design and Construction of Post-Tensioned Slabs-on-Ground**²⁰ specifies deflection ratios ranging from 1/240 to 1/2000 depending on the specifics of the construction and the assumed foundation distortion mode.
- Foundation repair contractors frequently warrant that any underpinned portion of the foundation will not settle more than 1/360 after the repair work is completed. This is equivalent to 1.33/480. For purposes of this paper, we will assume that the purpose of the analysis to develop a basis for recommending whether the foundation should be underpinned; thus the maximum foundation deflection ratio is taken to be 1/360.

In a center lift distortion pattern, the perimeter of the foundation sags downward. The corners typically sag the most as the supporting soil shrinks from both sides of the corners. If the short dimension of the rectangular model is 30-feet wide and the maximum deflection ratio for any deflection curve is 1/360, we assume that the entire foundation perimeter sags 1-inch and the corners sag another inch. Note that, in this case, the displacement at each corner is -2 inches. This deflection surface profile is illustrated in figure #1.

¹⁸ ACI Committee 435, 1972, *Variability of Deflections of Simply Supported Reinforced Concrete Beams*, Journal ACI, 69(1), p. 29

¹⁹ *Control of Deflection in Concrete Structures*, ACI 435-95, American Concrete Institute, p. 5

²⁰ **Design and Construction of Post-Tensioned Slabs-on-Ground**, Post-Tensioning Institute, Phoenix, Arizona, 1996

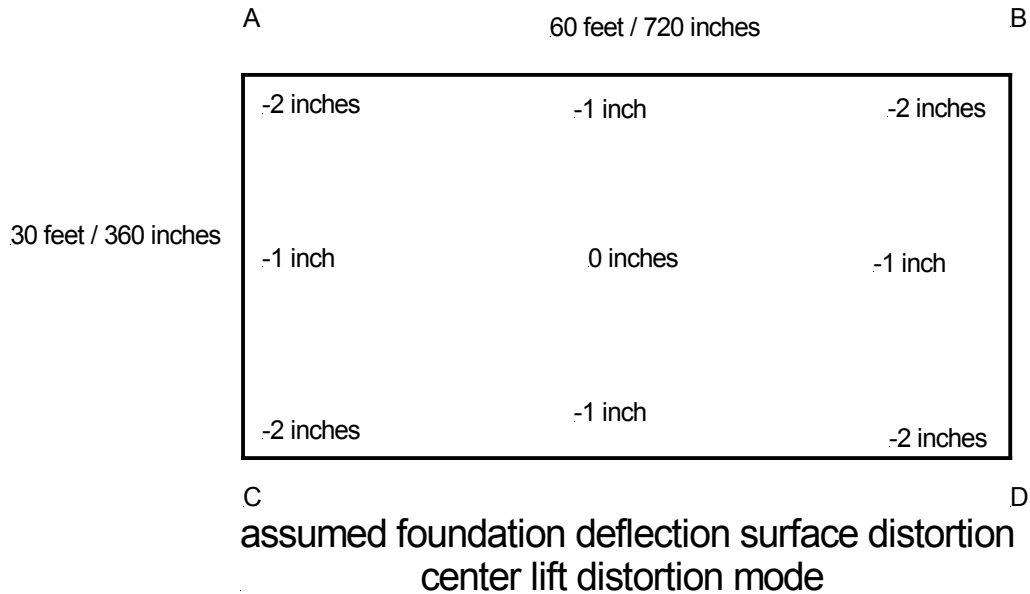


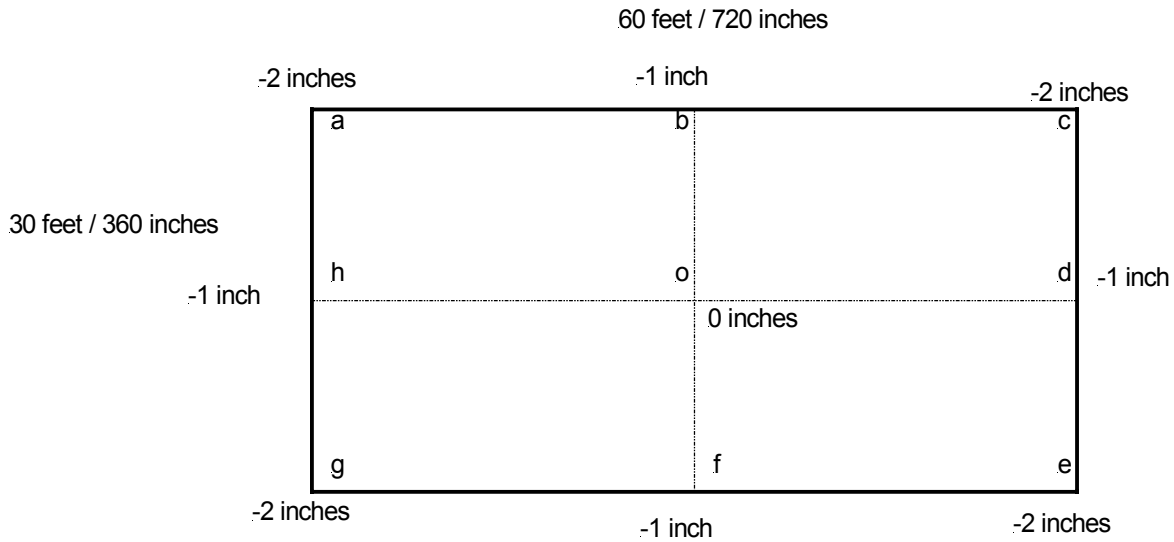
figure #1

Looking at figure #1, it is clear that the deflection ratio of deflection curves A-C and B-D is 1/360. Geometric compatibility demands that the 60-foot sides deflect to the same amount at points C, D, A and B. This means that the deflection ratio of the 60-foot sides is 1/720 or .5/360.

In summary, the deflection ratios of the perimeter deflection curves are given below:

deflection ratios for the perimeter deflection curves	
deflection curve	deflection ratio
A-B	1/720
B-D	1/360
C-D	1/720
A-C	1/360

In figure #2 below we have shown a plan view of the foundation with several deflection curves identified. In this sketch we have identified what I refer to as the primary deflection curves for the foundation.



assumed foundation deflection surface distortion
center lift distortion mode

figure #2

Note that deflection curves A-B, B-D, C-D and A-C have been split in half and replaced. Every deflection curve in the model has one end where the slope is zero while at the other end there is an unknown slope but a known deflection. These boundary conditions suggest that one way to model the primary deflection curves is to use a cantilever beam model. A cantilever beam has zero slope at the fixed end and both a slope and a deflection at the free end.

Below is a table showing the deflection of each primary deflection curve. The deflections come figure #1.

deflections for the primary deflection curves shown in figure #2			
deflection curve	deflection	deflection curve	deflection
a-b	1 inch	g-h	1 inch
b-c	1 inch	h-a	1 inch
c-d	1 inch	o-b	1 inch
d-e	1 inch	o-d	1 inch
e-f	1 inch	o-f	1 inch
f-g	1 inch	o-h	1 inch

The shape of the primary deflection curves is assumed to be that of a segment of a circle. This is a reasonable assumption since the foundation can be modeled as a long slender beam.²¹ Using a

²¹ **Mechanics of Materials**, 3rd edition, James M. Gere and Stephen P. Timoshenko, PWS-KENT Publishing Company, Boston, Massachusetts, p. 253

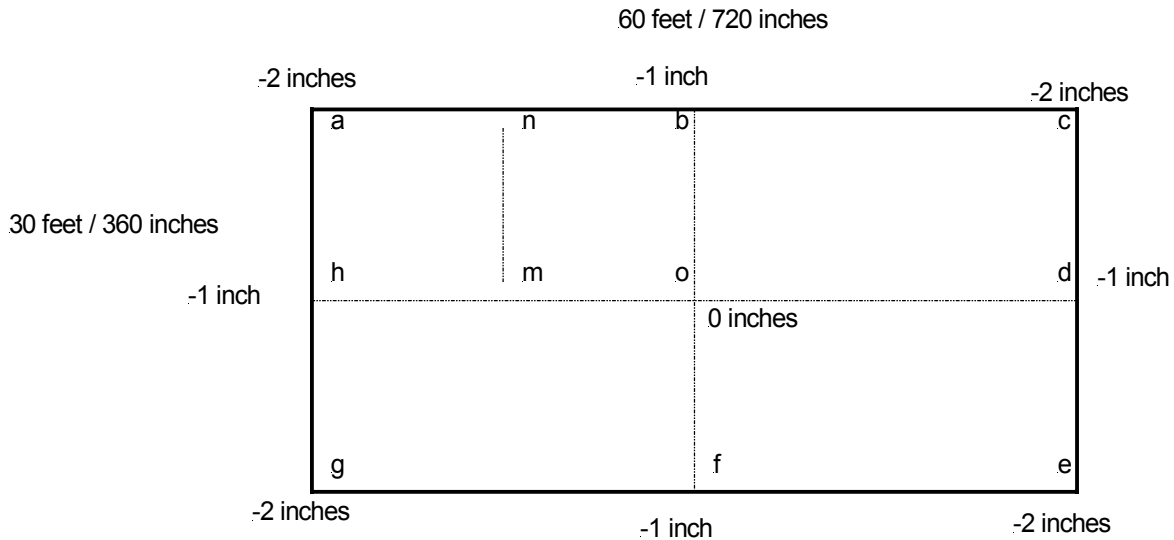
cantilever beam model in pure bending results in a deflection curve that is a segment of a circle since plane sections remain plane.

Consider deflection curve “b-c” in figure #2. The slope of deflection curve “b-c” at point “b” is zero and the deflection at “c” is 1 inch. Assuming that E, I and M are constant, the standard moment curvature equation, $EI(d^2v/dx^2) = M$, can be integrated twice and the boundary conditions of $v(L) = 1$ and $dv/dx(0) = 0$ applied to obtain the following equations for deflection and slope:

	at a distance x from the fixed end	at $x=L$ (L =the length of the beam model)
deflection (Δ)	$Mx^2/2EI$	$ML^2/2EI$
slope (θ)	Mx/EI	ML/EI

Taking the ratio of deflection to slope at $x=L$, we get $(\Delta/\theta_L) = L/2$ or $\theta_L = 2\Delta/L$ where Δ is the deflection at $x=L$ and L is the length of the cantilever beam. This allows us to calculate the slope at the free end with a known deflection at the free end. Since the curvature is circular, the slope changes linearly with x. Thus, since the slope at the fixed end is zero and the slope at $x = L$ can be calculated as $2\Delta/L$, the slope at any intermediate location can be calculated using linear interpolation. The calculated slope value at the location of interest (θ_x) can then be used to calculate the deflection using $\delta_x = \theta_x x/2$. Thus knowing the length and maximum deflection of any deflection curve allows us to calculate the slope and deflection at any point on the deflection curve. The only other characteristic of a primary deflection curve of interest is the radius of curvature, ρ . It can be shown that $\rho = L/\theta_L$. This procedure allows the calculation of the deflection and slope on any point on a primary deflection curve; also the radius of curvature can be calculated for any of the primary deflection curves.

We can now consider how the deflection and slope can be calculated for what I refer to as secondary deflection curves. Consider figure #3 below noting the addition of deflection curve labeled “m-n”. I refer to this deflection curve as a secondary deflection curve since the deflection and slope is determined by the intersecting primary deflection curves, in this case “a-b” and “o-h”.



assumed foundation deflection surface distortion
center lift distortion mode

figure #3

It is clear that the slope of deflection curve “m-n” at point “m” is zero. The deflection at point “n” must be calculated as the sum of the following:

- The deflection of deflection curve “o-b” at point “b” which in this case is 1 inch
- The deflection of deflection curve “a-b” at point “n” minus the deflection of deflection curve “o-h” at point “m”.

Since, for any given distance from the fixed end of the cantilever beam model, the deflection on deflection curve “a-b” is same as the deflection on deflection curve “o-h”, the deflection at point “n” of the deflection curve “m-n” is the same as the deflection of deflection curve “o-b” at point “b”. Thus, the deflection ratio of deflection curve “m-n” is the same as the deflection ratio of deflection curve “o-b”.

Recognizing this relationship makes it is possible to calculate the deflection and slope of any point on any secondary deflection curve consistent with the assumed center lift distortion mode.

Using Structural Mechanics to Analyze a Brick Veneer Wall

A brick veneer wall can be modeled as a deep beam that is the length, height and thickness of the brick veneer wall. In our model problem, there are four brick veneer walls; two are 30-feet long and two are 60-feet long. Therefore, we will examine a 30-foot wall and a 60-foot wall. Both walls will be assumed to be 8-feet high and 2.5 inches wide. Thus the 30-foot wall is modeled as a weightless elastic beam that is 30-feet in length, 8-feet in height and 2.5-inches in width and the 60-foot wall is modeled as a weightless elastic beam that is 60-feet in length, 8-feet in height

and 2.5-inches in width. For both walls, it is assumed that the neutral axis is at the center of the wall. If the elastic beam model predicts that the wall fractures, a single equivalent crack width can be estimated using an elastic beam model with the neutral axis at the bottom of the wall or by using a deflection-displacement diagram.

Since the beam model is relatively deep compared to the length, the contribution of shear deformation to the total deformation of the beam has to be accounted for. This can be done using the general equation for the maximum deflection of a simply supported elastic beam subjected to a uniform load as shown below:

$\delta = (5qL^4 / 384EI)(1 + 48f_s EI / 5GAL^2)$ <p>(the first term is the bending deflection and the second term is the shear deflection)</p>	<p>where q is a uniform load, E is the modulus of elasticity for the brick veneer wall, f_s is the form factor for the beam, I is the moment of inertia for the brick veneer wall, A is the cross sectional area of the brick veneer wall and G is the shear modulus of elasticity</p>
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This equation rests on the assumption that the total deflection of a deep beam is a superposition of shear and bending deflection.²²

Taking $E = 1,750,000$ psi and $G = .4E$, the maximum deflection due to pure shear and that due to pure bending can be calculated as follows assuming a deflection ratio of 1/360 for the 30-foot wall and a deflection ratio of 1/720 for the 60-foot wall. The results are shown in the following table.

maximum deflection due to bending and shear				
wall length	bending deflection		shear deflection	
	percentage	calculated	percentage	calculated
360 inches (30-feet)	85.4	.85 inches	14.6	.15 inches
720 inches (60-feet)	95.9	.96 inches	4.09	.041 inches

The critical strain is the strain at which cracking in a material becomes evident. In the procedure we use, the calculated bending, shear and combined strains are compared to the critical strains for brick veneer in order to determine if visible cracking is likely to occur for each deflection ratio being considered. The critical bending strain and shear strain in the table below are taken from published values. The critical combined strain is calculated using the standard equation from plane-strain mechanics²³ for the principle combined plane-strain as shown below where $\epsilon_y \approx 0$

²² **Mechanics of Materials**, 3rd edition, James M. Gere and Stephen P. Timoshenko, PWS-KENT Publishing Company, Boston, Massachusetts, p. 253

²³ **Mechanics of Materials**, 3rd edition, James M. Gere and Stephen P. Timoshenko, PWS-KENT Publishing Company, Boston, Massachusetts, p. 253

$$\text{combined principle strain: } \varepsilon_{combined} = \frac{\varepsilon_x}{2} + \sqrt{\left(\frac{\varepsilon_x}{2}\right)^2 + \left(\frac{\lambda_{xy}}{2}\right)^2}$$

The calculated bending strain is calculated using the radius of curvature assuming deflection due to bending-only and the neutral axis at the mid height of the wall ($\varepsilon_{bending} = H/2\rho$); the calculated shear strain is calculated using the shear-only deflection and assuming that the deflected shape of the wall is circular. This gives a maximum shear strain of $\lambda_{shear} = 4\Delta/L$. The calculated combined strain is calculated using the standard equation for the principle combined plane-strain as shown below where $\varepsilon_x = \frac{\varepsilon_{bending}}{2}$, $\varepsilon_y \approx 0$ and $\lambda_{xy} = \lambda_{shear}$.

critical strains	
critical bending strain	.0005
critical shear strain	.0015
critical combined strain	.000791

In order to calculate the appropriate combined strain, the location on the beam model must be selected for the calculation. The selection of an appropriate combined strain location must be based on a rational understanding of the stress distribution in beams. Since the only load supported by the deep beam model is the weight of the brick veneer wall, it is obvious that an appropriate model is a simply supported beam subjected to a uniform load. The maximum tensile bending stress is clearly located at outermost fiber at the top of the wall. There is no bending stress at the neutral axis. The maximum shear stress is at a maximum near the ends of the wall at the neutral axis; the shear stress vanishes at the outermost fibers. Thus, it is not clear where the maximum combined principle stress and strain will be located.

Based on calculated stress trajectory curves and stress contours, the maximum combined principle stress and strain is likely to be at or near the ends of the wall and about half way between the neutral axis and the top outer most fiber.²⁴

Based on the above understanding of the combined stress and strain distribution in the beam, we use half of $\varepsilon_{bending}$ and the full value of the shear strain. This results in the following equation for calculating the maximum combined strain.

$$\text{calculated combined strain: } \varepsilon_{combined} = \frac{\varepsilon_{bending}}{4} + \sqrt{\left(\frac{\varepsilon_{bending}}{4}\right)^2 + \left(\frac{\lambda_{shear}}{2}\right)^2}$$

The results for our example walls are given below:

²⁴ **Mechanics of Materials**, 3rd edition, James M. Gere and Stephen P. Timoshenko, PWS-KENT Publishing Company, Boston, Massachusetts, p. 423

critical and calculated strains for 30-foot and 60-foot walls			
	critical strain	calculated strain	
		30-foot wall	60-foot wall
bending strain	.0005	.00253	.000710
shear strain	.0015	.00162	.000810
combined strain	.000791	.00229	.000797

It is clear that, for our example, all of the brick veneer walls will fracture; brick veneer is simply very weak in tension and this virtually guarantees that a brick veneer wall will fracture under even small deflections.

Once the wall fractures, the cracking is likely to follow an irregular diagonal stair-step pattern through the mortar, mortar/brick interface and through brick units along the principle stress trajectories. The cracking pattern will be influenced by discontinuities in the brick veneer such as at windows and doors. There are several ways to estimate the equivalent single crack width. If the wall fractures mainly in bending, the single equivalent crack width can be estimated using an elastic beam model with the neutral axis at the bottom of the beam and multiplying the bending strain by the length of the wall. If shear deflection is a significant portion of the total deflection, the calculated combined strain can be used. In using combined strain, engineering judgment must be carefully exercised in selecting the distance over which the combined strain should be applied.

In some cases the brick veneer wall may break apart and the pieces rotate as rigid panels. This can be analyzed using a deflection-displacement diagram. For instance, if we assume that the wall breaks into two equal length panels and that they each rotate through the average slope of the deflection curves, the single equivalent crack width (at the top of the wall) would be given by the following equation where ECW is the estimated crack width, H is the height of the brick veneer wall, and Δ/L is the deflection ratio using the total deflection:

$$ECW = 4H\left(\frac{\Delta}{L}\right)$$

The following table shows results of estimated equivalent single crack width calculations for the 30-foot and 60-foot walls.

wall length	estimated equivalent single crack width at the top of the wall		
	deformation diagram for rigid panel rotation	elastic beam bending only	elastic beam combined strain applied across the wall diagonal
30-foot	1.06 inches	.918 inches	.853 inches
60-foot	.533 inches	.511 inches	.579 inches

Engineering judgment must be exercised in selecting the model to use when estimating the equivalent single crack width. For our example wall, if we assume that the brick veneer is a normal running bond, the best model is a bending-only elastic beam model.

Using Structural Mechanics to Analyze the Rotation of a Fireplace Chimney

There is no question that foundation deflection can result in a masonry fireplace chimney rotating away from the house. Considering that fireplace chimneys are commonly located at the perimeter of the foundation makes it almost inevitable that the fireplace chimney will rotate away from the house structure in a center lift distortion mode. A deflection-displacement diagram showing a fireplace chimney rotating away from the house is shown in the figure below:

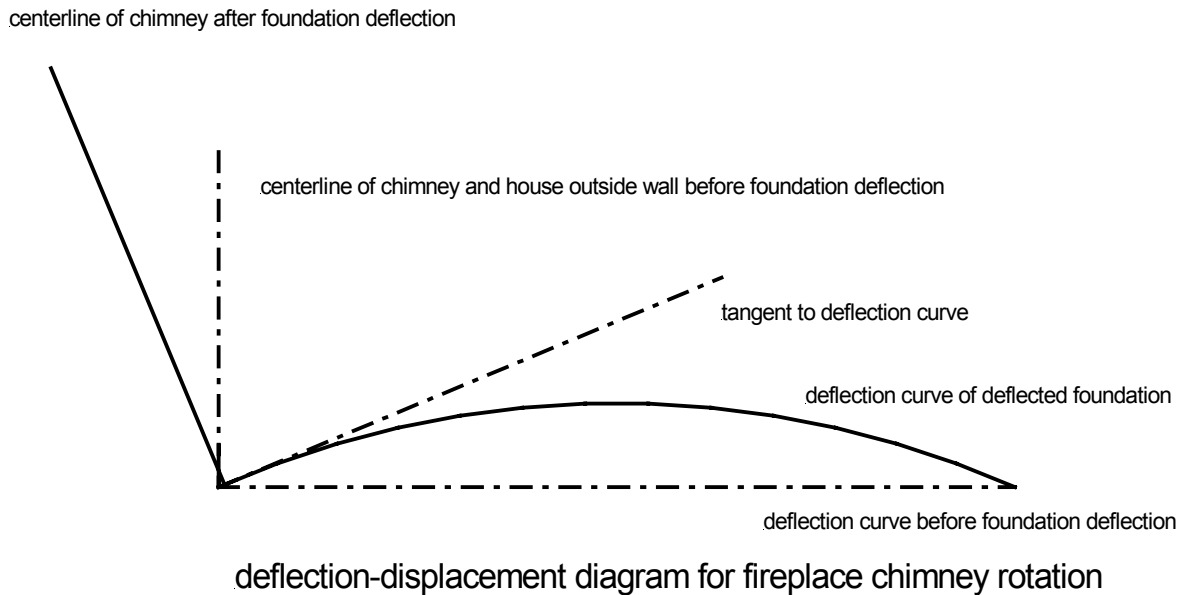
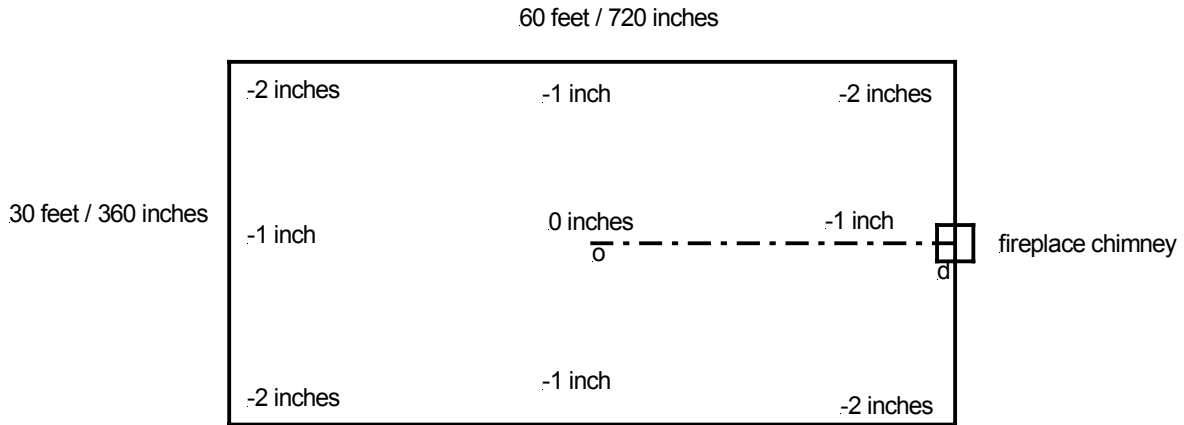


figure #4

Referring to figure #5, the relevant deflection curve, “o-d”, can be modeled as a cantilever beam that is 30 feet in length in pure bending and deflecting 1 inch at the free end.



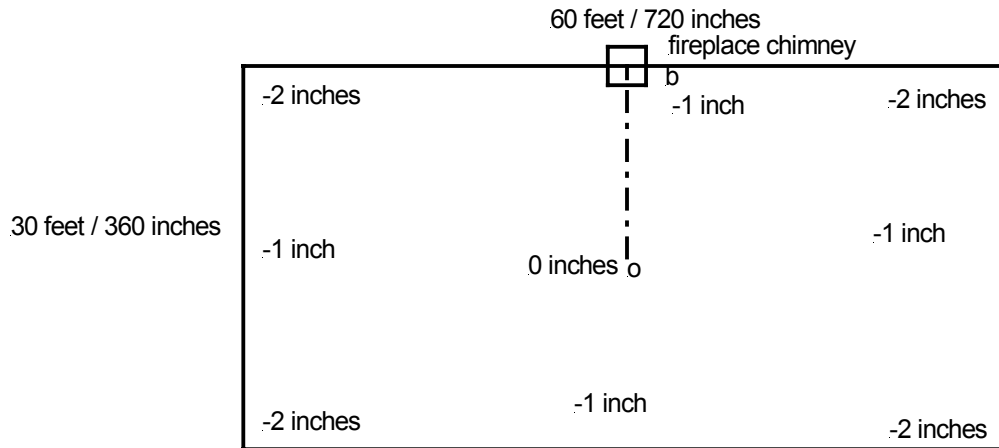
assumed foundation deflection distortion showing
the deflection curve "o-d" and the fireplace chimney

figure #5

The slope of the deflection curve at the free end (given by $2\Delta/L_{o-d}$) would be .00556. The gap between the fireplace chimney and the roof at the roof intersection can be estimated by multiplying the slope by the height of the roof/fireplace chimney intersection above the foundation deflection surface.²⁵ This implies a gap between the roof and the fireplace chimney of .600 inches assuming the roof is 9-foot above the foundation deflection surface.

As a second example, consider a fireplace chimney that is located at point "b" on deflection curve "o-b" as shown in figure #6 below:

²⁵ cf. Meyer, Kirby T., *Defining Foundation Failure*, Texas Section, ASCE, Padre Island, Fall 1991



assumed foundation deflection distortion showing the deflection curve "o-b" and the fireplace chimney

figure #6

The slope at point "b" on deflection curve "o-b" is .0111. This gives a gap of 1.2 inches at a fireplace chimney/roof intersection 9-feet above the above foundation deflection surface.

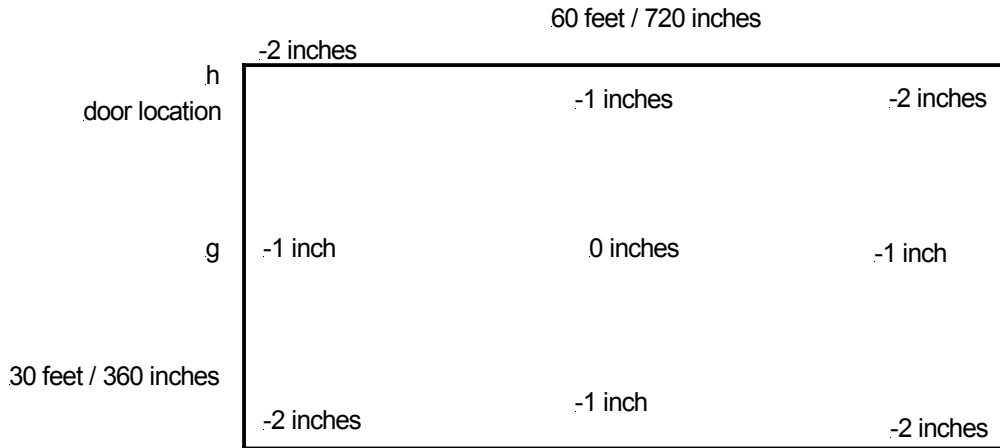
Using Structural Mechanics to Analyze Door Frame Distortion

As a slab-on-ground foundation bends, the wood frame structure reacts by racking. A door frame prior to the foundation movement would be reasonably square if normal construction practices were used. The door frame will be forced out of square as the slab floor and the top of the door frame move in response to the change in the slope of the deflection curve at the location of the door.²⁶

The first step in calculating whether a door will bind is to calculate the slope of the deflection curve at the location of the door on the deflection curve. We have already seen how the slope is calculated. The next step is to determine the critical gap through which the door frame must displace for the door to bind. If a door is hinged on the left and the deflection curve slopes upward from left to right, the critical gap will be the gap between the door and the floor. This is normally around 1/2 inch. Assuming the door frame to be 32-inches wide, the critical slope would be the critical gap divided by the width of the door frame. In our example, assuming that the critical gap is .5 inch, the critical slope would be $.5/32$ or .0156. On the other hand, if the same door were hinged on the right side, the critical gap would be the gap between the door and the door frame. This is normally around 1/8th inch. In this case the critical slope would be $(1/8)/32$ or .00391.

We can illustrate this approach by considering the door shown in the following plan view of our foundation.

²⁶ Meyer, Kirby T., *Defining Foundation Failure*, Texas Section, ASCE, Padre Island, Fall 1991

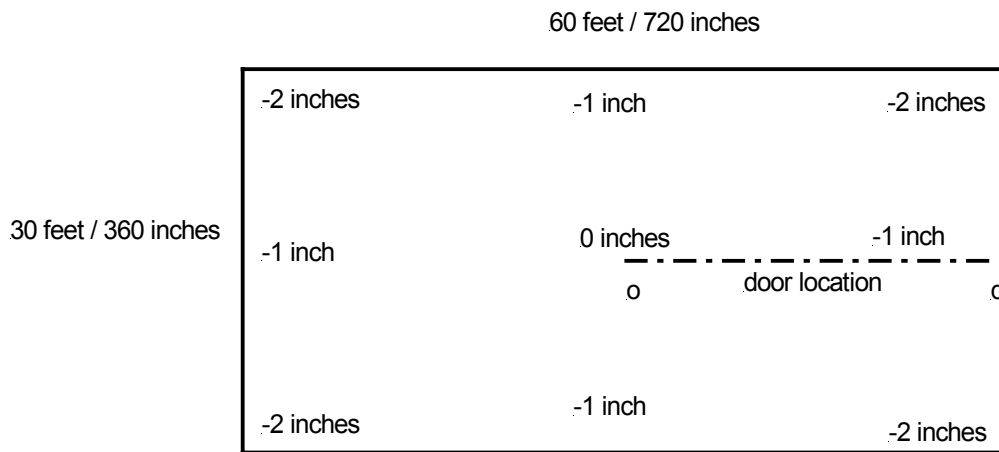


assumed foundation deflection surface distortion showing the deflection curve "g-h" and the door location at the end the deflection curve

figure #7

The center of the door is assumed to be located 24-inches from point “h” on deflection curve “g-h”. The deflection curve is modeled as a cantilever beam fixed with no slope at “g” and deflecting 1 inch at “h”. If the door is hinged at the “h” end, the critical slope would be .00391 assuming a 32 inch wide door and a 1/8th gap between the door and the door frame at the top of the door. The slope at the end of the deflection curve, is given by $2\Delta/L_{g-h}$ where L_{g-h} is 180 inches. This gives a slope of .0111. The slope at the door location can be calculated as .00962. Clearly the model would predict that the door would bind. Let us now consider the same door but hinged on the other side so that the critical slope would be .0156. Since the slope of the deflection curve at the door location is only .00962, the model predicts that the door will not bind.

Now we can consider a second door located on the deflection curve labeled “o-d” midway between the center of the foundation and the right end as shown below in figure #8:



assumed foundation deflection distortion showing the deflection curve "o-d" at the door location at the middle of the deflection curve

figure #8

This deflection curve can be modeled as a cantilever beam fixed at “o” and deflecting 1 inch at “d”. We will further assume that the door is hinged on the side of the door closest to the center of the foundation. At this location, the slope of the deflection curve is .00278. Assuming that the critical gap at the top of the door is 1/8th inch and the width of the door is 32-inches, the critical slope is .00391. In this case the model predicts that the door will not bind.

Using Structural Mechanics to Analyze Gypsum Board Cracking

The analysis of gypsum board cracking in wood frame walls is very similar to the analysis of door frame distortion. As the foundation deflects, the curvature will cause the wall frame to rack. The most common location for a gypsum board crack to develop is at the corner of a door frame or window frame. The critical value of angular distortion for gypsum board is reported to be between .37% and .7%²⁷. We use the average value of .54% or .0054. This critical angular distortion is for a sheet of gypsum board without any stress concentrations. The openings in the gypsum board at doors and windows clearly create stress concentrations at these locations. It is difficult to say what the stress concentrations are quantitatively, but they are undoubtedly high, probably at least 2.5.²⁸ Thus, the critical angular distortion adjusted for the presence of stress

²⁷ Bozozuk, M. (1962). *Soil Shrinkage Damages Shallow Foundations at Ottawa, Canada*, The Engineering Journal, July 1962, Canada, pp. 33-37

²⁸ Walter D. Pilkey (1997). **Peterson’s Stress Concentration Factors**, 2nd edition, John Wiley and Sons, Inc., New York, New York

concentrations is taken to be .0054 divided by 2.5 or .00216. The following calculations are based on the same doors used in the section on door frame distortion.

If we look at the door located 24-inches from point “h” on deflection curve “g-h”, the slope at the door location was previously calculated as .00962. Clearly, the angular distortion as measured by the slope of the deflection curve door location exceeds the critical angular distortion even if we do not adjust it for the presence of stress concentrations. Thus, the model predicts that the gypsum wall board will fracture at this door location.²⁹

We can now consider the door in the middle of deflection curve “o-d”. The slope of the deflection curve at this door location was previously calculated as .00278. This is more than the stress concentration adjusted critical angular distortion; therefore the model predicts that the wall board at this door location will fracture.

Using this approach, the gypsum wall at any door, doorway or window can analyzed to determine if the gypsum wall board will crack.

Using Structural Mechanics to Make Engineering Judgments Concerning Underpinning Slab-on-Ground Foundations

The following table is a damage matrix for a brick veneer home. It shows how the estimated damage could be reported.

Damage Matrix for a Brick Veneer Single Family Home				
brick veneer walls	visible crack/separation strain			
	calculated	points	observed	points
wall A-B (60-feet long)				
wall B-D (30-feet long)				
wall C-D (60-feet long)				
wall A-C (30-feet long)				
fireplace chimney rotation	calculated gap		observed gap	
door frame distortion	number calculated		number observed	
gypsum board cracks	number calculated		number observed	
	total calculated points		total observed points	

²⁹ it is interesting that the structural mechanics models we are using predict that gypsum drywall at a door location will crack with less angular distortion that that required to cause the door to stick; this is consistent with the experience of many engineers

The general procedure for constructing a damage matrix like this consists of the following steps:

- Using the procedures described above, each brick veneer wall is analyzed to see if it cracks and to estimate the equivalent single crack width.
- The gap between the roof and any perimeter fireplace chimney intersection is calculated using the procedure described above
- Calculations are made for each hinged door to see if the assumed foundation distortion will cause the door frame to distort enough to make the door stick or bind
- Every door, doorway and window opening is analyzed to see if the angular distortion or the wall frame will cause the drywall to fracture

The observed damage to the house is likely to be a disparate mix of several forms of distortion. In order to compare the estimated or calculated damage to the observed damage, we need a way of reducing the calculated and observed distortions to a single quantity or index. We can do this by assigning points to the distortions. The following is a point system that we have found useful:

point system for comparing estimated damage to observed damage	
visible cracking in the brick veneer walls	1 point for each 1/8-inch of visible strain
fireplace chimney rotation	1 point for each 1/8 th inch gap at 9-feet above the slab-on-ground foundation surface
door frame distortion	2 points for each sticking or binding door
gypsum board cracks	1 point for each drywall fracture at a door, doorway or window

In the above matrix, the calculated points are the points that correspond to the calculated damage to the house that could result from the curvature of the slab-on-ground foundation for a specified deflection ratio. In this case the deflection ratio used is 1/360. This represents the damage to the house that could result after the foundation was underpinned if future movement matched the warranty, which we assume to be 1/360. The observed damage point total represents the damage that was observed at the time of the inspection. If the calculated points exceed the observed points, underpinning may not make a significant improvement to the performance of the foundation unless the underpinned foundation performed better than the contractor is willing to warrant. On the other hand, if the observed damage clearly exceeds the calculated damage, underpinning might be a viable option.

There are other factors that should be taken into account when recommending whether a foundation should be underpinned.

Conclusions Regarding the Use of Structural Mechanics in Residential Foundation Performance Evaluation

The purpose of this paper, as stated above, was to demonstrate the use of structural mechanics to develop logical and rational deflection/damage assessment procedures for the purpose of making a quantitative first-order assessment of the damage that could be caused by center lift deflection of a slab-on-ground foundation. By "rational" I refer to the fact that the analysis relies only on engineering structural mechanics and published laboratory results of material tests. Generally, the procedures described in this paper allow the following:

- consideration of building and building element geometry
- consideration of critical strain properties of building materials
- consideration of probable slab-on-ground foundation distortion patterns and geometry
- calculation of quantifiable results using accepted structural engineering models, concepts and tools
- calculated results are in forms that bear clear physical meaning that can be readily understood by a typical homeowner or buyer

This paper has shown that fundamental structural mechanics can be used to develop idealized mathematical models that can be used to relate geometric changes in slab-on-ground foundations to specific types of damage in the supported structure. These engineering models can be used to generate a rational basis for making building damage assessments and foundation performance evaluations. The paper demonstrates the use of simple structural engineering models such as deep beam deformation, strain superposition, plane-strain mechanics and deflection-displacement analysis in evaluating whether a slab-on-ground foundation should be underpinned. This approach is a logical extension of past work in the area of ground and foundation movement induced building damage.

Like any other approach to foundation performance evaluation, the approach described in this paper has both advantages and disadvantages. Some of these are listed below:

Disadvantages:

- The approach is measurement intensive compared to a visual inspection or a procedure based on an elevation survey. The measurements, however, can be made with an ordinary tape measure.
- The structural mechanics approach requires that the evaluating engineer be able to make reliable engineering judgments based on a thorough understanding of fundamental structural mechanics including deep beam theory, engineering beam theory, plane-strain mechanics, deflection/displacement mechanics and strain superposition.

- The approach is calculation intensive. It is not unusual for a house to require hundreds of calculations. On the other hand, the procedure can be easily adapted for spreadsheet calculations.
- Primarily because of the need for a large number of measurements, the structural mechanics approach is more time consuming. Since the calculations are made using a computer spreadsheet template, most of the time required by an evaluation is spent taking measurements and in preparing the report.

Advantages:

- The structural mechanics approach provides a repair criterion in the form of a damage index that is not based on a single generalized criteria such as angular distortion. The use of a single generalized repair criteria such as crack width or elevation differences involves inherent simplifications (such as ignoring building geometry and critical strains of the wall coverings) that are more likely to result in less reliable engineering judgments concerning the wisdom of underpinning the foundation.
- The procedure is based on accepted structural mechanics models that provide a realistic description of the damage foundation movement can cause a house. Yet, the models use simple building and foundation deformation geometry that allows damage to the building to be easily calculated.
- Familiarity with the use of the approach described in this paper will make an engineer able to make more reliable judgments concerning foundation performance when making a visual inspection.³⁰
- The structural mechanics approach can be used to make estimates of damage a house could experience in the future. This is not possible to do in a rational and logical way using an elevation survey approach.
- The structural mechanics approach does not rely on foundation surface elevation measurements thus obviating the necessity to take two sets of elevation measurements or estimating as-built foundation out of levelness.
- The structural mechanics approach is very flexible. It can be adopted to accommodate any reasonable deflection ratio, foundation shape or distortion pattern.

³⁰ I should add that most homes do not require the type of analysis described in this paper. In my experience, most homes require only a visual inspection for an experienced and competent structural engineer to render a reliable opinion as to the need to underpin the foundation. Engineers should take cognizance of Rule 131.155(4) of the Texas Board of Professional Engineers which prohibits the performance of unnecessary work.